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On unification of RR couplings

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Abstract: We consider the couplings of RR fields with open string sector for Dp-$\overline{\text{D}p}$ backgrounds of various p. The proposed approach, based on the approximation of the open string algebra by the algebra of differential operators, provides the unified description of these couplings and their interrelations.

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1. Introduction

The process of brane annihilation [1] leads to the unified description of the backgrounds with various D-brane configurations [2]. However, despite the recent progress in understanding the tachyon low energy action and the related qualitative phenomena [3]–[6], the picture of the dynamics of the D-brane formation/annihilation is still not very clear. One of the most important points seems to be the interrelation of the open and closed string sectors in this process, which is one of the main mysteries of string theory. The satisfactory understanding of this relation most likely will provide the clue regarding the symmetries of string theory [6].

In this paper we reconsider the couplings of the RR gauge fields of the closed string sector with the fields of the open string sector. These couplings [7] are especially simple due to their anomalous origin [8] and thus provide a suitable framework for the discussion of the various open string backgrounds. These couplings were extensively studied (see e.g. [9]–[20]). Our ultimate goal will be the unified description of these couplings for various $D-\overline{D}$ backgrounds.[1] The picture that emerges could be considered as an explicit realization of some proposals from [21] (some related remarks have been made in [17]).

[1] We were informed by G. Moore that some related studies have been undertaken by him in collaboration with Duiliu-Emanuel Diaconescu.

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In an attempt to provide the unified description of the RR gauge field couplings for various background brane configurations it is natural to consider the off-shell interpolation of these backgrounds. The connection with K-theory uncovered in [2, 22] leads to the description of the RR-gauge field couplings in terms of the superconnection [23]. Thus a natural framework for the universal description of RR couplings would be in terms of a superconnection in some universal (infinite) dimensional bundle $\mathcal{E}$ over space-time M which provides interpolation between various brane/anti-brane configurations. This infinite-dimensional bundle $\mathcal{E}$ should be naturally connected with the geometry of M and thus it is not very surprising to find the "tautological" bundle with infinite-dimensional fiber isomorphic to space of functions on the base manifolds. This bundle has a rich structure connected with the action of the differential operators in the fiber. This is in perfect agreement with the appearance of the differential operators in the explicit description of the elements of the K-homolgy groups.

The appearance of the differential operators is also natural from string field theory point of view. Configuration space of the open string theory is roughly given by the maps of the interval into space-time. If we approximate the strings by straight lines the configuration space reduces to space of the pairs of the points (ends of the intervals) in space-time. Therefore the functionals on configuration space become the functions of two points and could be interpreted as the kernels of the integral operators. The expansion around diagonal (image of the interval is a point) leads naturally to the differential operators.

In this note we discuss very briefly the approximation of the open string algebra by the differential operators. The more systematic exposition will is given in [24]. Here we describe explicit interpolation of the various RR gauge field couplings using the formalism of superconnections. For simplicity we always consider the case of the flat space-time. The generalizations to curved manifolds are rather obvious. In the process of describing our proposal we will focus on the local properties.

The organization of this paper is as follows. In section 2 we consider the interpolation between the RR gauge field couplings with open strings for various brane/anti-brane backgrounds in terms of superconnections acting in appropriate finite-dimensional ($\mathbb{Z}_2$-graded) bundles. The physical interpretation of the construction is through the appearance of D-branes of higher codimension in the process of annihilation of D$-\overline{\text{D}}$-branes with the corresponding non-trivial low-dimensional RR charges. In order to get an arbitrary brane configuration one could use only D-branes of the highest possible dimension. For simplicity we will illustrate these phenomena looking at type-IIB theory and D9$-\overline{\text{D9}}$ system [2]. It should be noted that the configurations with non-equal number of D9- and $\overline{\text{D}}$9-branes is anomalous in string theory. But in our approach to RR-couplings all D-branes are more or less on equal footing and we will not discriminate the case of D9-branes in the following presentation.

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As an application we derive explicit formula for the topological density of the "fat" D5-brane inside the D9-brane, which coincides with the well-known formula for the densities of the instanton charge (see e.g. [25]). The main deficiency of the approach of this section is the necessity to consider special configurations of the D9$-\overline{\text{D9}}$-branes depending on the desired ultimate configuration of the lower-dimensional brane/anti-brane system we would like to get. The lesson learned from these examples suggests the idea to search for the universal description by taking the number of D9$-\overline{\text{D9}}$-branes infinite [21] (see [14] for earlier discussions).[2]

Our main proposal is presented in section 3. Here we describe the concrete procedure of taking the limit by considering the superconnection in the appropriate geometrically defined infinite-dimensional bundle. We show how the explicit formulas from the section 2 appear in this formalism. At the end we briefly discuss the interpretations of the proposed universal description in terms of string theory. The issues related to the appearance of non-abelian Chan-Paton factors in the proposed universal description and its connection with background independent open string field theory [28, 29] will be published separately [24].

2. RR-couplings from the finite number of D9 – $\overline{\text{D9}}$-branes

In this section we consider the RR gauge field couplings with open strings for the backgrounds with the finite number D9$-\overline{\text{D9}}$-branes and the low-dimensional brane backgrounds arising in the process of the annihilation.

The general coupling of the D-brane fields with the RR fields may be characterized as follows. Any configuration of D-branes in type-IIB theory defines some element of the K^0-group of space-time [2, 22] which may be represented as a formal difference of the vector bundles up to some equivalence. This was interpreted in [2] as gauge bundles on D9- and $\overline{\text{D9}}$-branes filling whole space-time. There is a canonical map of the K^0-group to the cohomology of the manifold defined with the help of the Chern character. In order to get explicit representation of this map in terms of the (closed) differential forms one should supply the difference of the bundles with an appropriate analog of the connection — superconnection [23]. The superconnection corresponding to a given configuration of D9$-\overline{\text{D9}}$-branes may be constructed in terms of the open string field. The coupling of the RR gauge fields with open string sector of the D9$-\overline{\text{D9}}$-branes filling whole space-time has the following form:

$$S_{\text{RR}} = \mu_9 \int_{M^{(10)}} [C_{\text{RR}} \wedge \text{Ch}(\mathbb{A})]_{\text{top}} \, . \tag{2.1}$$

[2]Another motivation for this comes from the search of closed string in the field theory of open strings [26]- N D1-branes thought as multi-soliton on space-filling D9-brane, or as D9-compactified on T^8, in large-N limit and IR describes type-II closed string via Matrix String construction [27].

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Here μ_9 is the D9- and $\overline{\text{D9}}$-brane charge, $M^{(10)}$ is ten-dimensional flat target space, $\text{Ch}(\mathbb{A})$ is the Chern character of the superconnection $\mathbb{A}$ constructed from the open string modes, C_{RR} is defined as a sum of the RR gauge fields of the given parity (even in IIB theory):

$$C_{\text{RR}} = \sum_k C_{(2k)} \,, \qquad C_{(2k)} = C_{\mu_1 \cdots \mu_{2k}}\, dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_{2k}} \tag{2.2}$$

and the subscript *top* means that we should integrate the top-dimensional differential form over $M^{(10)}$. We use the normalization of RR gauge fields consistent with the standard definition of the superconnection used below.

The gauge bundles on D9 and $\overline{\text{D9}}$-branes may be combined in a $\mathbb{Z}_2$ graded bundle with the superconnection defined in terms of the gauge field A_μ on the D, the gauge field $\widetilde{A}_\mu$ on the $\overline{\text{D}}$ and the complex tachyon field T corresponding to the lowest energy excitation of the strings stretched between D and $\overline{\text{D}}$ by the following:

$$\begin{aligned} \mathbb{A} &= \begin{pmatrix} dx^\mu \nabla^+_\mu & T \\ \overline{T} & dx^\mu \nabla^-_\mu \end{pmatrix}, \\ \nabla^+_\mu &= \partial_\mu + \mathrm{i}\, A_\mu \,, \qquad \text{and} \qquad \nabla^-_\mu = \partial_\mu + \mathrm{i}\, \widetilde{A}_\mu \,. \end{aligned} \tag{2.3}$$

The Chern character of the superconnection is defined by the standard formula:

$$\text{Ch}(\mathbb{A}) = \text{Str}\; e^{-\mathbb{A}^2/2\pi} \,. \tag{2.4}$$

Here we use the supertrace on the matrices acting on $\mathbb{Z}_2$-vector spaces which is defined as follows. Let $V = V_+ \oplus V_-$ be a $\mathbb{Z}_2$-vector space and $\mathcal{O} \in \text{End}(V)$:

$$\begin{pmatrix} \mathcal{O}_{++} & \mathcal{O}_{+-} \\ \mathcal{O}_{-+} & \mathcal{O}_{--} \end{pmatrix} . \tag{2.5}$$

Then the supertrace of $\mathcal{O}$ is defined as:

$$\text{Str}_V\; \mathcal{O} = \text{Tr}_V\; \tau\, \mathcal{O} = \text{tr}_{V_+}\; \mathcal{O}_{++} - \text{tr}_{V_-}\; \mathcal{O}_{--} \,, \tag{2.6}$$

where τ is an operator defining $\mathbb{Z}_2$-structure.

In the following subsections we consider various families of the background open string fields and show how the RR gauge field couplings for the lower-dimensional D-branes appear in some limits.

2.1 D-instantons from D9 − $\overline{\text{D9}}$-branes

We start with the simple case of k D-instantons inside ten-dimensional flat space, which appear as the result of the D9−$\overline{\text{D9}}$ annihilation. In order to construct k D-instantons from the D9−$\overline{\text{D9}}$ system one has to consider $16\,k$ D9-branes and $16\,k$ $\overline{\text{D9}}$-branes [1, 2, 30], where 16 Chan-Paton indexes from both sides are embedded into the spinor bundles of the target space with opposite chirality.

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The corresponding superconnection may be described as follows [2]. Consider a $\mathbb{Z}_2$-graded vector bundle $V = V_+ \oplus V_-$, where $V_\pm = S_\pm \otimes E_k$ and $S_\pm$ are the spinor bundles of the definite chirality and E_k is a k -dimensional vector bundle. The superconnection is defined as in (2.3) with:

$$\begin{aligned} \nabla^+ &: \; V_+ \longrightarrow V_+ \\ \nabla^- &: \; V_- \longrightarrow V_- \\ T &: \; V_+ \longrightarrow V_- \\ \overline{T} &: \; V_- \longrightarrow V_+ \end{aligned} \tag{2.7}$$

We take a trivialization of the vector bundle E_k, i.e. $\nabla^\pm = d$.

To begin we describe the simplest situation of coincident D-instantons which are left after the D9$-\overline{\text{D9}}$-annihilation. The corresponding superconnection may be constructed as the limit $t \to 0$ of the superconnection (2.3) with:

$$\nabla = d\,, \qquad \text{and} \qquad T = \frac{1}{\sqrt{t}}\, x^\mu\, \sigma_\mu \otimes \mathbf{1}_{k\times k}\,, \tag{2.8}$$

where σ_μ are ten-dimensional γ-matrices in the Majorana-Weyl representation and $\mathbf{1}_{k\times k}$ is the identity matrix acting in the space E_k (k is the number of the instantons). Description of $\mathrm{Ch}(\mathbb{A})$ in terms of such a family of superconnections is a standard procedure and appeared in mathematical literature long ago [23]. In the language of string theory it may be interpreted as off-shell interpolation (one shall compare this tachyon profile T with the one from [2]). The family (2.8) corresponds to the smearing of the D-instanton charge over a region of the size $\propto \sqrt{t}$ (see eq. (2.13) below).

The square of the superconnection in question has the form:

$$\mathbb{A}_t^2 = \begin{pmatrix} \frac{1}{t}\,|x|^2 & \frac{1}{\sqrt{t}}\, dx^\mu\, \sigma_\mu \\ \frac{1}{\sqrt{t}}\, dx^\mu\, \overline{\sigma}_\mu & \frac{1}{t}\,|x|^2 \end{pmatrix} \otimes \mathbf{1}_{[k\times k]} = \left(\frac{1}{\sqrt{t}}\, dx^\mu\, \gamma_\mu + \frac{1}{t}\,|x|^2 \right) \otimes \mathbf{1}_{[k\times k]}\,. \tag{2.9}$$

Taking into account that in the spinor notations supertrace of the arbitrary operator $\mathcal{O}$ may be represented in terms of the trace over spinor representation, Sp, as

$$\mathrm{Str}\,\mathcal{O} = \mathrm{tr}_k\ \mathrm{Sp}\ \gamma^{11}\mathcal{O}\,, \tag{2.10}$$

we have:

$$\mathrm{Ch}(\mathbb{A}_t) = k\ \mathrm{Sp}\ \gamma^{11} e^{-\mathbb{A}_t^2/2\pi}\,. \tag{2.11}$$

Here $k = \mathrm{tr}\,\mathbf{1}_{[k\times k]}$. Thus, using the identities for d-dimensional space:

$$\mathrm{Sp}\,\gamma^{d+1}\,\gamma^{\mu_1}\cdots\gamma^{\mu_d} = (2i)^{d/2}\epsilon^{\mu_1\cdots\mu_d} \tag{2.12}$$

and

$$\lim_{t\to 0} \frac{1}{(\pi t)^{d/2}}\ \exp\left\{-\frac{|y|^2}{t}\right\} = \delta^{(d)}(y)\,, \tag{2.13}$$

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we obtain:

$$\mathrm{Ch}(\mathbb{A}) = \lim_{t\to 0} \mathrm{Ch}(\mathbb{A}_t) = k\,\delta^{(10)}(x)\ \mathrm{vol}(M^{10})\,. \tag{2.14}$$

After the substitution into (2.1) this gives $S_{\mathrm{RR}} = k\,\mu_{-1}\,C_{(0)}(x=0)$, i.e. the correct coupling of k D-instantons with RR gauge fields.

Consider a more general situation given by the deformation of the tachyon profile (2.8) as follows:

$$T = \frac{1}{\sqrt{t}}\,(x^\mu\ \gamma_\mu \otimes \mathbf{1}_{k\times k} - \Phi^\mu\,\gamma_\mu)\,, \tag{2.15}$$

with Φ^μ — constant $k\times k$ hermitean matrix, $\mu = 0,\dots,9$, which capture the fluctuations of the relative positions of k D-instantons. These matrices naturally appear in the low energy descriptions of the D-instantons in string theory [31]. For the square of the superconnection we have:[3]

$$\mathbb{A}_t^2 = \frac{1}{\sqrt{t}}\,dx^\mu\,\gamma_\mu \otimes \mathbf{1}_{k\times k} + \frac{1}{2t}\,[\Phi^\mu,\,\Phi^\nu]\gamma_\mu\,\gamma_\nu + \frac{1}{t}\,|x^\mu \otimes\ \mathbf{1}_{k\times k} - \Phi^\mu|^2. \tag{2.16}$$

Let us define the symmetric trace "SymTr" following [33, 34] as the symmetrisation of the usual trace. Thus we have:

$$\begin{aligned}\mathrm{Tr}\,e^{A+B+C} &= \mathrm{Tr}\sum_n \frac{(A+B+C)^n}{n!} = \mathrm{SymTr}\sum_n \frac{(A+B+C)^n}{n!}\\ &= \mathrm{SymTr}\,e^{A+B+C}\,.\end{aligned} \tag{2.17}$$

Taking into account (2.16) and (2.17) we obtain:

$$\begin{aligned}S_{\mathrm{RR}} &= \mu_{-1}\int_{M^{(10)}} C_{\mathrm{RR}}(x)\ \mathrm{SymTr}\ e^{-[i_\Phi,\ i_\Phi]/2\pi}\,\delta^{(10)}(x\otimes\mathbf{1}^\mu - \Phi^\mu)\\ &= \mu_{-1}\int_{M^{(10)}} C_{\mathrm{RR}}(x)\ \mathrm{SymTr}\ e^{-[i_\Phi,\ i_\Phi]/2\pi}\,e^{-\mathrm{i}\Phi_\mu\partial_\mu}\,\delta^{(10)}(x)\\ &= \mu_{-1}\ \mathrm{SymTr}\ e^{-[i_\Phi,\ i_\Phi]/2\pi}\,C_{\mathrm{RR}}(\Phi)\,,\end{aligned} \tag{2.18}$$

where i_Φ is defined in [34] — the indexes of Φ's in the commutator in the exponent are contracted with those of $C_{(2k)}$. We also use the following definition of the δ-function:

$$\delta^{(d)}(x^\mu\otimes\mathbf{1} - \Phi^\mu) = \lim_{t\to 0}\frac{1}{(\pi t)^{d/2}}\ \exp\left\{-\frac{1}{t}\sum_{\mu=1}^{d}|x^\mu\otimes\mathbf{1} - \Phi^\mu|^2\right\}. \tag{2.19}$$

Note that the expression (2.18) coincides with S_{RR} obtained in [34].

One could consider more general deformations of T:

$$T \propto x^\mu\,\sigma_\mu\otimes\mathbf{1} + T_{(k)}^{\mu_1\cdots\mu_k}(x)\,\sigma_{\mu_1\cdots\mu_k} + \cdots\,, \tag{2.20}$$

which apparently correspond to massive open string excitations.

[3] Similar formula appears in the context of SW map, as was communicated to us by N. Nekrasov, see [32, eq. (42)].

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2.2 Dp-branes from the D9 $-$ $\overline{\text{D9}}$

It is not hard to generalize the above construction to the case of k Dp-branes with $p < 9$. Let us separate indexes $\mu = 0, \ldots, 9$ into $m = 0, \ldots, p$ and $i = p+1, \ldots, 9$ and consider the following background values of the tachyon and gauge field:

$$\nabla = d\,, \qquad \text{and} \qquad T = \frac{1}{\sqrt{t}}\, x^i\, \sigma_i \otimes \mathbf{1}_{k\times k}\,. \tag{2.21}$$

Here now σ_i are $(9-p)$-dimensional σ-matrices. Now, in order to construct k Dp-branes from the D9$-\overline{\text{D9}}$ system, one has to consider [2] $2^{(9-p)/2} \times k$ D9-branes and the same number of $\overline{\text{D9}}$-branes.

Similarly to the case of the instantons the superconnection (2.21) leads to the coupling:

$$S_{\text{RR}} = k\mu_p \int_{M^{(p+1)}} C_{(p+1)}(x_m)\,,$$

which is the source corresponding to k Dp-branes; $M^{(p+1)}$ here is the world-volume of the Dp-branes.

Let us consider the fluctuations around (2.21) of the form:

$$\begin{aligned} &\nabla = [\partial_m + \mathrm{i}\, A_m(x_m)]\, dx^m + \partial_i\, dx^i\,, \\ \text{and} \qquad &T = \frac{1}{\sqrt{t}}\left[x^i\ \sigma_i \otimes \mathbf{1}_{k\times k} - \Phi^i(x_m)\,\sigma_i\right]. \end{aligned} \tag{2.22}$$

Here A and Φ are the gauge field and scalar field living on the Dp-brane world volume.[4] The explicit expression for the square of the superconnection is:

$$\begin{aligned} \mathbb{A}_t^2 \;=\; &[\nabla_m,\ \nabla_n]\, dx^m\, dx^n + \frac{1}{\sqrt{t}}\, dx^i\, \gamma_i \otimes \mathbf{1}_{k\times k} + \frac{1}{\sqrt{t}}\,[\nabla_m,\ \Phi_i]\gamma^i\, dx^m + \\ &+ \frac{1}{2t}\,[\Phi^i,\ \Phi^j]\gamma_i\,\gamma_j + \frac{1}{t}\,|x^i \otimes \mathbf{1}_{k\times k} - \Phi^i|^2\,. \end{aligned} \tag{2.23}$$

After substitution of this expression into (2.1) and taking $t \to 0$, one finds:

$$S_{\text{RR}} = \mu_p \int_{M^{(p+1)}} \left[\text{SymTr}\; C_{\text{RR}}(x_m,\, \Phi_i)\, e^{-\frac{1}{2\pi}\left(F_{(2)}(x_m) + [\nabla_{(1)}(x_m),\ i_\Phi] + [i_\Phi,\ i_\Phi]\right)}\right]_{\text{top}}, \tag{2.24}$$

with $F_{(2)} = [\nabla_m,\, \nabla_n]\, dx^m \wedge dx^n$ and $\nabla_{(1)} = \nabla_m\, dx^m$. This expression also coincides with S_{RR} considered in [34].

[4] Note that while the scalars Φ_i appear from the tachyon field, the gauge field A_m in this case originates from the gauge field localized on the D9$-\overline{\text{D9}}$-system. In the next section we will consider the "gauge equivalent" description where A_m will take its origin from the tachyon field as well.

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2.3 General configuration of D-branes of various codimensions

The general case is now rather obvious. For illustration we will construct a very simple configuration of low-dimensional D-branes. Consider D-branes oriented along coordinate hyperplanes in d =10-dimensional flat space. One describes such a configuration with k_p Dp-branes and $\widetilde{k}_p$ $\overline{\mathrm{D}p}$-branes for all p by considering the collection of D9$-\overline{\mathrm{D9}}$ which represents $\mathbb{Z}_2$-graded bundle:

$$V = V_+ \oplus V_- = \oplus_p (E_{k_p} \oplus E_{\widetilde{k}_p}) \otimes S(9-p)\,. \tag{2.25}$$

Here E_{k_p} and $E_{\widetilde{k}_p}$ are vector bundles of dimension k_p and $\widetilde{k}_p$ and $S(9-p)$ are the spinor bundles in the directions transverse to the Dp-branes ($S(0)$ is a trivial $\mathbb{Z}_2$-even bundle). The sum in (2.25) is taken over the odd p's (we are considering type-IIB string theory). $\mathbb{Z}_2$-grading is given by the operator τ acting on spinor bundles $S(9-p)$ as $\tau = (-1)^{p/4}\,\gamma^{10-p}$ where γ^{10-p} is a product of all gamma-matrices acting in $S(9-p)$ (analog of γ^5 in $(9-p)$ dimensions). It acts on the bundles E_{k_p} and $E_{\widetilde{k}_p}$ as follows:

$$\begin{aligned} \tau E_{k_p} &= E_{k_p}\,, \\ \tau E_{\widetilde{k}_p} &= -E_{\widetilde{k}_p}\,. \end{aligned} \tag{2.26}$$

Then the obvious generalization of the superconnection (2.22) is:

$$\begin{aligned} \mathbb{A}_{t,\widetilde{t}} = \nabla + \sum_p \frac{1}{\sqrt{t_p}} \sum_{i=p+2}^{10} (x^i\ \gamma_i \otimes \mathbf{1}_{k_p\times k_p} - \Phi^i\,\gamma_i) + \\ + \sum_p \frac{1}{\sqrt{\widetilde{t}_p}} \sum_{i=p+2}^{10} (x^i\ \gamma_i \otimes \mathbf{1}_{\widetilde{k}_p\times \widetilde{k}_p} - \widetilde{\Phi}^i\,\gamma_i)\,. \end{aligned} \tag{2.27}$$

Here the matrices $\mathbf{1}_{k_p\times k_p}$ ($\mathbf{1}_{\widetilde{k}_p\times \widetilde{k}_p}$) acts as unit matrices on E_{k_p} ($E_{\widetilde{k}_p}$) and by zero otherwise. It gives rise to the density of the Chern character which in the limit $t_p, \widetilde{t}_p \to 0$ describes the coupling of the collection of the Dp-branes with the RR fields.

The general superconnection (2.27) interpolates between various configurations of D-branes capturing all their low-energy fluctuations. Note an interesting novel feature of the general case. The general deformations of this superconnection includes operators transforming in the *spinor* representations of the transversal Lorentz groups. These tachyonic excitations correspond to the lowest energy modes of the strings stretched between the Dp- and $\overline{\mathrm{D}p}$-branes with the mixed Neumann-Dirichlet boundary conditions. They are responsible for the "thickening" of the lower-dimensional D-branes inside bigger D-branes. In the next subsection we proceed with the discussion of some example of this situation.

2.4 D5-branes inside D9-branes and the instanton charge density

Consider the special case of the k D5-branes inside N D9-branes as the result of the D9$-\overline{\mathrm{D9}}$ annihilation. The theory with $N > 0$ is anomalous. However, this is not

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important for our further considerations and we proceed with this case in order to make the presentation most transparent. Otherwise one could consider D3−D7-brane system using the machinery of the previous subsection.

The $\mathbb{Z}_2$-graded vector bundle in this case has the form:

$$V = V_- \oplus V_+ = (W_N^{(9)} \ \oplus E_k^{(5)} \otimes S(4)_-) \oplus (E_k^{(5)} \otimes S(4)_+)\,, \tag{2.28}$$

where $S_\pm$ are definite chiral, Weyl fermion bundles in four-dimensional space transverse to the D5-branes. We take a trivialization of the $E_k^{(5)}$ bundles $\nabla = d$.

In order to make a comparison with on-shell description in terms of the instanton moduli space we restrict the superconnection under consideration by the condition that it is invariant with respect to the natural action of Sp(1)-group on S_+. Then its moduli space is parameterized by ADHM data (see e.g. [35]):

$$\begin{aligned} T &= \frac{1}{\sqrt{t}}\Delta\,, \\ \Delta &\ : \ S_-(4) \otimes E_k^{(5)} \oplus W_N^{(9)} \longrightarrow S_+(4) \otimes E_k^{(5)}\,. \end{aligned} \tag{2.29}$$

Here Δ is the standard ADHM matrix [35]:

$$\Delta = [x_i \ \sigma^i \otimes \mathbf{1}_{k\times k} + B_i(x_m)\,\sigma^i,\ h(x_m)]\,, \tag{2.30}$$

where $i = 6, \ldots, 9$ are directions transversal to the D5-brane and $x_i\,\sigma^i$ naturally acts as the operator $x : S_- \to S_+$, similarly constant (independent of x_i) operator B acts as $B : S_- \otimes E_k \to S_+ \otimes E_k$ and the constant operator h acts as $h : W_N \to S_+ \otimes E_k$. In the language of string theory B_i's are scalars localized on the D5-brane world-volume, which appear as the massless excitations of the strings with both their ends on the D5-brane. They parameterize transversal fluctuations of the D5-brane. In these terms h appears as the massless excitations of strings stretched between D5- and D9-branes.

The condition on the matrix Δ in ADHM construction is equivalent to invariance with respect to the action of Sp(1)-group on S_+ and thus we cover the whole ADHM moduli space. The derivation of this tachyon soliton from the condition of SUSY restoration after the annihilation was given in [36].

The Chern character of the superconnection $\mathbb{A}_t$ at the limit $t \to 0$ reduces to the standard expression for the Chern character of the instanton solutions. Consider for the illustration the simple case of one instanton in the Sp(1) gauge theory: $k = 1$, $\dim W = 2$. It is useful to identify $W \cong S_+$. Choose the configuration of the instanton with the center at $x = 0$ and of the radius h. Then the corresponding superconnection is:

$$\mathbb{A}_t = d \otimes \mathbf{1}_{4\times 4} + \frac{1}{\sqrt{t}}\left[x^i\,\gamma_i \oplus 0_W + h p_w + h\ p_w^*\right]. \tag{2.31}$$

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Here $B = 0$ and $p_w : W \to S_+$ identifies W and S_+. Note that $\frac{1-\gamma^5}{2} p_w = p_w$ and $\frac{1+\gamma^5}{2} p_w = 0$. The square of the superconnection is:

$$\mathbb{A}_t^2 = \frac{1}{\sqrt{t}}\, dx^i\, \gamma_i \oplus 0_W + \frac{1}{t}\left[|x|^2\ \mathbf{1}_S + h^2\, \mathbf{1}_{S_+} + h^2\, \mathbf{1}_W + h\, x^i\, \sigma_i\, p_W + h\, p_W^*\, x^i\, \overline{\sigma}_i\right], \quad (2.32)$$

where we use the notations: $\mathbf{1}_S$ is the projector on the spinor subspace, $\mathbf{1}_{S_+} = \frac{1+\gamma^5}{2}$ is the projector on the S_+ subspace and $\mathbf{1}_W$ is the projector on W. Thus, in matrix form, such $\mathbf{A}$ superconnection looks as:

$$\mathbb{A}_t^2 = \begin{pmatrix} \frac{1}{t}\left(|x|^2 + h^2\right) & \frac{1}{\sqrt{t}}\, dx^i\, \sigma_i & 0 \\ \frac{1}{\sqrt{t}}\, dx^i\, \overline{\sigma}_i & \frac{1}{t}\, |x|^2 & \frac{1}{t}\, h\, x^i \sigma_i \\ 0 & \frac{1}{t}\, x^i\, \overline{\sigma}_i\, h & \frac{1}{t}\, h^2 \end{pmatrix}. \quad (2.33)$$

One can use the identity:[5]

$$e^{A+B} = \mathrm{P}\ \exp\left\{\int_0^1 dt\, e^{-t\,B}\, A e^{t\,B}\right\} e^B \quad (2.34)$$

and obvious commutation relations between γ_i, $\mathbf{1}_S$, $\mathbf{1}_{S_+}$, $\mathbf{1}_W$ and p_w, in order to expand the expression $\exp(-\mathbb{A}_t^2)$ in the components having the differential forms of the definite degree and prove that:

$$\mathrm{Ch}(\mathbb{A}) = \lim_{t\to 0} \mathrm{Sp}\ \mathrm{tr}\,(\gamma^5 \oplus \mathbf{1}_W)\, e^{-\frac{\mathbb{A}_t^2}{2\pi}} \propto \dim(W) + \frac{1}{8\pi}\frac{h^4}{(|x|^2 + h^2)^4}\ \mathrm{vol}(\mathbb{R}^4). \quad (2.35)$$

This is the correct expression for the topological charge density of one Sp(1) YM instanton [25]. The more conceptual derivation of this formula and its generalizations to the multiple instantons with higher range gauge groups using the results of [37] will be published elsewhere [38].

3. Universal description and algebra of differential operators

The construction described in the previous section has an obvious drawback. Auxiliary (super)-vector bundles that appear in the description of D-branes do not have clear geometrical meaning. In particular for each D-brane configuration one needs to consider the specific collection of vector bundles. On the other hand it is obvious that different vector bundles with the same topological invariants lead to the equivalent description of the D-branes. In order to get a universal description of D-brane backgrounds it is natural to look at the bundles of the infinite rank [21]. We will use

[5] In order to prove this formula consider "log"-derivative over t of the operator $G(t) = e^{t(A+B)} e^{-t\,B}$:

$$G(t)^{-1} d_t G(t) = e^{-t\,B}\, A\, e^{t\,B}.$$

The integration over t gives the desired identity.

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a concrete realization of this limiting bundle as the bundle where fiber is identified with the space of functions (more exactly sections of the finite-dimensional bundles) on an auxiliary space. As an auxiliary space it appears natural to take a copy of the space where string theory is defined. The idea to use this realization of the universal bundle comes from simple qualitative arguments concerning the configuration space of the open strings that were discussed in the introduction. Note that the natural algebra acting on the fiber of this bundle is the algebra of differential operators on auxiliary space.

This construction may be considered as an application of the general approach of [24] to the description of the RR gauge field couplings.

In the next subsection we show how to derive the corresponding RR field coupling for the D9-brane filling the whole space-time. Then we proceed with the derivation of the RR gauge field coupling for an arbitrary case. This reproduces the expressions considered in section 2.

3.1 Description of D9-branes

We start the description of the universal construction with simple physical motivations. Consider $16\,k$ D9-branes and $16\,k$ $\overline{\text{D9}}$-branes in the flat space -time $M = \mathbb{R}^{10}$ and take $k \to \infty$. In this way we construct $k \to \infty$ D($\overline{\text{D}}$)-instantons. We could use these D($\overline{\text{D}}$)-instantons to construct various Dp-branes. The configuration of k non-coincident D-instantons may be parameterized by k points of space-time. More correctly it is the space of 10 mutually commuting $k \times k-$matrices up to conjugation. It is easy to see that as the number k of D-instantons tends to infinity the k-dimensional vector space on which these matrices act becomes more and more like Hilbert space of the square integrable functions on space-time M where string theory is defined. In these terms the matrices themselves may be considered as differential operators (more generally integral operators) acting on functions on M. The traces of matrices in this limit may be reduced to the integrals over the space M. For any operator function $F(\widehat{q},\widehat{p})$ of the coordinates $\widehat{q}^{\mu}$ and momentum $\widehat{p}_{\mu}$ one has:

$$\mathrm{Tr}_{\mathcal{H}} : F(\widehat{q},\widehat{p}) := \int dp \wedge dq \, \langle q| : F(\widehat{q},\widehat{p}) : |p\rangle \, \langle p|q\rangle = \int dp \wedge dq \, F(q,p)\,, \qquad (3.1)$$

where $: F :$ is the normal ordering of the operators such that the momentum operators act on the right and the position operators act on the left.

Note that to have a well-defined trace the operators should have special property (to be of the trace class). A particular class of such operators is given by the expressions $F(\widehat{p},\widehat{q}) \propto \delta^{(10)}(\widehat{p}) f(\widehat{q})$. It's trace is naturally represented by the integral over the lagrangian submanifold $M \in T^*M$ rather than over T^*M and thus the operators of this kind and its smooth deformations are natural candidates for the limits of the appropriate matrix variables. This type of operator has a simple qualitative interpretation in terms of string backgrounds. Let us symbolically denote open string by the

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matrix $K_{x,y}$ (or more exactly integral operators $\widehat{K}$ with the kernel $K(x,y)$ acting on the space of functions on M) where possible end points of the strings are enumerated by the indexes "x" and "y". The trace invariants naturally correspond to the open strings with identified ends and thus may be considered as closed strings. Now let us consider the trace of the operator $\widehat{K}$ with additional insertion of the operator $\delta(\widehat{p})$ ($\widehat{p}$ is momentum operator) which projects on the states with zero eigenvalue of $\widehat{p}$. It is easy to see that:

$$\delta(\widehat{p}) = |p=0\rangle \langle p=0| = \left(\sum_x |x\rangle\right)\left(\sum_{x'} \langle x'|\right) \tag{3.2}$$

(note that the identity operator is $1 = \sum_x |x\rangle \langle x|$) and we have:

$$\mathrm{Tr}_{\mathcal{H}}\ \delta(\widehat{p})\widehat{K} \sim \sum_{x,x'} K(x,x')\,. \tag{3.3}$$

We see that in the presence of the operator we should sum over the positions of the "ends" of the string independently. Thus the insertion of the operator $\delta(\widehat{p})$ may be interpreted as the creation of the D-brane filling the whole space-time.

We have replaced the consideration of the infinite number of D-branes by the consideration of the infinite-dimensional vector bundles over the space-time M with the fiber — the space of functions on the copy $\widetilde{M}$ of M. The Lie algebra of the differential operators acts naturally in the fibers of the bundle. Note that one may consider infinite-dimensional bundles with the fiber given by the sections of a finite-dimensional (super)-bundle. We encounter examples with the space of sections of spin bundles as a fiber.

Now we give an explicit construction of the superconnection in this bundle which leads to the desired description of RR-couplings. Let us start with few remarks on the notations. In order to distinguish the space-time M from the auxiliary space $\widetilde{M}$ we will use the coordinates (x^μ) on $M \equiv M_x$ and the coordinates (y^μ) on $\widetilde{M} \equiv M_y$. Thus the matrices realize the quantization of the space of functions on T^*M_y and the corresponding Hilbert space may be constructed in terms of the sections of the spinor bundle over M_y.

Having in mind more general cases treated below we consider Clifford algebra for the total space $M_x \otimes T^*M_y$:

$$\begin{aligned} &\{\gamma_\mu, \gamma_\nu\} = \delta_{\mu\nu}\,, \qquad \{\Gamma_\mu, \Gamma_\nu\} = \delta_{\mu\nu}\,, \qquad \{\widehat{\Gamma}^\mu, \widehat{\Gamma}^\nu\} = \delta^{\mu\nu}\,, \\ &\{\Gamma_\mu, \gamma_\nu\} = \{\widehat{\Gamma}^\mu, \Gamma_\nu\} = \{\widehat{\Gamma}^\mu, \gamma_\nu\} = 0\,. \end{aligned} \tag{3.4}$$

The gamma matrices are defined with respect to the explicit coordinates as follows: $(x^\mu, y^\nu, p_\rho) \leftrightarrow (\gamma_\mu, \Gamma_\nu, \widehat{\Gamma}^\rho)$.

First we give a simple example of the superconnection describing the D9-brane filling the whole space-time. Taking into account the heuristic description in terms of the infinite number D-instantons considered at the beginning of this subsection

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and the results of the first part of the paper we could propose the following superconnection:

$$\mathbb{A}_{s,t} = d + \frac{1}{\sqrt{t}}\,\gamma_\mu(x^\mu - y^\mu) + \frac{1}{\sqrt{s}}\,\widehat{\Gamma}^\mu\,\frac{\partial}{\partial y^\mu}\,. \tag{3.5}$$

This superconnection takes values in the space of differential operators acting in the auxiliary space of sections of spinor bundle on M_y with standard $\mathbb{Z}_2$-structure defined by the chirality. The first term is a direct analog of the tachyonic profile (2.15). Note also that the last term is a Dirac operator on this auxiliary space.

The superconnection (3.5) corresponds to the tachyon field of the following form:

$$T\,(x|\;y,\widehat{p}) = \frac{1}{\sqrt{t}}\,\sigma_\mu(x^\mu - y^\mu) + \frac{1}{\sqrt{s}}\,\widehat{\Sigma}^\mu\widehat{p}_\mu\,, \tag{3.6}$$

where $\widehat{p} = \partial/\partial y$. The first term is rather obvious and one of the reasons for introducing the last term is to fulfill the condition of being trace class.

The square of (3.5) is given by the expression:

$$\mathbb{A}^2_{s,t} = \frac{1}{t}\,|x-y|^2 + \frac{1}{s}\,|\widehat{p}_\mu|^2 + \frac{1}{\sqrt{st}}\,\gamma_\mu\,\widehat{\Gamma}^\mu + \frac{1}{\sqrt{t}}\,dx^\mu\,\gamma_\mu \tag{3.7}$$

and the Chern character is given by:

$$\mathrm{Ch}(\mathbb{A}) = \lim_{t\to 0}\lim_{s\to 0}\mathrm{Ch}(\mathbb{A}_{s,t})\,. \tag{3.8}$$

Note that here "Str" is taken over the representation of the full Clifford algebra (3.4) and includes $\mathrm{Tr}_{\mathcal{H}}$ as well. By the trivial considerations it may be reduced to the following trace over the space of y-dependent functions:

$$\mathrm{Ch}(\mathbb{A}) = \mathrm{Tr}_{\mathcal{H}}\;\delta^{(10)}\,(\widehat{p}^\mu)\;\delta^{(10)}(x-y) = 1\,. \tag{3.9}$$

Thus we have reproduced the correct coupling with the top-degree RR-form for the D9-brane filling whole space-time: $S_{\mathrm{RR}} = \mu_9\,\int_{M^{(10)}} C_{(10)}(x)$.

One could include a non-trivial connection over y-variables. Let us show that this is equivalent to the inclusion of the same connection on M_x. Consider the following superconnection:

$$\mathbb{A}_{s,t} = d + \frac{1}{\sqrt{t}}\,\gamma_\mu\,(x^\mu - y^\mu) + \frac{1}{\sqrt{s}}\,\widehat{\Gamma}^\mu\,(\partial_\mu + \mathrm{i}\;A_\mu(y))\,. \tag{3.10}$$

Simple calculation gives:

$$\begin{aligned}\mathrm{Ch}(\mathbb{A}) &= \mathrm{Tr}_{\mathcal{H}}\left[\delta^{(10)}(\partial_\mu + \mathrm{i}\;A_\mu)\,\delta^{(10)}(x-y)\;\exp\left\{-\frac{1}{2\pi}F_{(2)}[A(y)]\right\}\right]\\ &= \exp\left\{-\frac{1}{2\pi}F_{(2)}[A(x)]\right\}.\end{aligned} \tag{3.11}$$

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At the last step we use the following identity: let us represent δ-function as:

$$\delta^{(10)}\left(\partial_\mu + \mathrm{i}\, A_\mu(y)\right) = \int dq\, e^{\mathrm{i}\, q^\mu (\partial_\mu + \mathrm{i}\ A_\mu(y))} \tag{3.12}$$

and use:

$$e^{\mathrm{i}\, q^\mu (\partial_\mu + \mathrm{i}\ A_\mu(y))} = e^{\mathrm{i}\, q^\mu\, \partial_\mu}\, e^{-\int_0^1 A_\nu(y + t\, q) q^\nu\, dt}\,. \tag{3.13}$$

this follows from (2.34). Furthermore, for an arbitrary function $F(p, y)$, we have

$$\begin{aligned}\mathrm{Tr}_{\mathcal{H}} \int dq\, e^{\mathrm{i}\, q^\mu\, \partial_\mu}\, F(q, y) &= \int dp\, dy\, dq\, \langle y | e^{\mathrm{i}\, q^\mu\, \partial_\mu} | p \rangle\, \langle p | F(q, y) | y \rangle \\ &= \int dp\, dq\, dy\, e^{\mathrm{i}\, q^\mu\, p_\mu} F(q, y) = \int dy\, F(0, y)\,. \end{aligned} \tag{3.14}$$

This proves (3.11). Thus:

$$S_{\mathrm{RR}} = \mu_9 \int_{M^{(10)}} \left[C_{\mathrm{RR}} \wedge e^{-\frac{1}{2\pi} F_{(2)}[A]} \right]_{\mathrm{top}}\,, \tag{3.15}$$

which is what we expect for D9-branes. In conclusion, we could equivalently turn on the gauge fields in x-space and in y-space.

3.2 Towards Universal Description of Dp-brane RR-couplings

In this subsection we give a general construction of the RR gauge field couplings for arbitrary D-brane background.

Let us start with a fixed infinite-dimensional bundle over the space-time with the fiber identified with the space of sections of the spinor bundle over the base space times the two-dimensional vector space. We start with the construction of a superconnection corresponding to n_+ D7-branes and n_- of $\overline{\mathrm{D}}7$ parallel branes. Let $(y_a = y_9, y_8)$ be coordinates in the orthogonal plane and $W(y_9, y_8)$ is a function defined by the condition that its only critical points are the positions of D7- and $\overline{\mathrm{D}}7$-branes in the plane (y_9, y_8) (the sign of the Hessian $\partial^2 W$ defines the sign of the RR charge). Consider now the superconnection:

$$\mathbb{A}_{s,t} = d + \frac{1}{\sqrt{t}}\, \gamma_\mu\, (x^\mu - y^\mu) + \frac{1}{\sqrt{s_1}}\, \Gamma_a\, \partial^a W(y) + \frac{1}{\sqrt{s_2}}\, \widehat{\Gamma}_\mu\, \widehat{p}^\mu\,. \tag{3.16}$$

In this formula the additional two-dimensional vector space is interpreted as the representation of the Clifford algebra of the cotangent space to the transverse space (the notations are in agreement with (3.4). Note that this representation is very close to the expression that enter the Supersymmetric Quantum Mechanic interpretation of Morse theory [39].[6] In fact, the calculation of the Chern character for this connection

[6] One could say that we have $N = 1$ SUSY Quantum mechanic in the directions orthogonal to D-brane and $N = 1/2$ SUSY Quantum mechanic in the directions parallel to D-brane

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in the limit $t, s_i \to 0$ gives the expression with the insertion of the projector in the y-integral:

$$\det(\partial_a \partial_b W)\, \delta^{(2)}(\partial_a W(y)) \tag{3.17}$$

along with $\delta^{(10)}(x-y)$ and $\delta^{(10)}(\widehat{p})$. Trivial calculations reduce the full expression to the one given in the first part of the paper and gives rise to the Chern character form representing n_+ D7-branes n_- $\overline{\text{D}}$7-branes ($n_\pm$ is number of critical points of $W(y)$ with the positive and negative indexes).

Note that at the intermediate step after taking the limit $s_1 \to 0$ we get an infinite-dimensional bundle with the fiber naturally identified with the space of sections of the super bundle of spinors on the auxiliary space of two dimensions lower multiplied by the finite-dimensional super-bundle $E = E_+ \oplus E_-$ with $\dim E_\pm = n_\pm$. Let us represent the ten-dimensional space-time $\mathbb{R}^{10}$ in the factorized form $\mathbb{R}^{10} = \mathbb{R}^2 \otimes \mathbb{R}^{8-p-1} \otimes \mathbb{R}^{p+1}$. One could identify the bundle E with the bundle of the representations of Clifford algebra of the cotangent space to $\mathbb{R}^{8-p-1}$ (similar to identification of the gauge bundle and tangent bundle in string compactifications). Now we could again use the same localization procedure. Suppose we would like to get the superconnection leading to the RR-coupling with Dp parallel branes orthogonal to $\mathbb{R}^2 \otimes \mathbb{R}^{8-p-1}$. Let $W^{(p)}(y)$ be a function on $\mathbb{R}^{8-p-1}$ whose critical points define the positions of Dp-branes. Then the following superconnection leads to the RR coupling with Dp-branes:

$$\begin{aligned}\mathbb{A}_{s,t} = d + \frac{1}{\sqrt{t}}\,\gamma_\mu\,(x^\mu - y^\mu) + \Bigg(\Gamma_a\,\Bigg(\partial^a\,\Bigg(\frac{1}{\sqrt{s_1}} W(y_9, y_8) + \\ + \frac{1}{\sqrt{s_p}} W^{(p)}(y_7, \ldots, y_{p+2})\Bigg)\Bigg)\Bigg) + \\ + \frac{1}{\sqrt{s_2}}\,\widehat{\Gamma}_\mu\,\widehat{p}^\mu\,.\end{aligned} \tag{3.18}$$

Here the summation is over $a = 9, 8, \ldots, (p+2)$.

Iteration of this procedure leads to the expressions for RR gauge field couplings discussed at the first part of the paper. The set functions $W^{(p)}$ emerged in this iterative procedure is rather special and connected with the special geometry of D-brane configurations. Consideration of arbitrary functions leads to the expression for RR gauge field coupling with an arbitrary configuration of D$-\overline{\text{D}}$-branes.

4. Conclusions

In this note we have proposed a universal description of the couplings of RR gauge fields with open strings in arbitrary backgrounds of the $\text{D}p - \overline{\text{D}p}$ for $p < 9$ in terms of the superconnection on some naturally defined infinite-dimensional bundle. This description is given in terms of the differential operators in the auxiliary space (which

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is a kind of a double of genuine space-time where string theory is defined) depending on the point of space-time. From the mathematical point of view this construction is very close to Beilinson construction of the holomorphic bundles on $\mathbb{P}^n$ [40].

In conclusion let us give some comments on the interpretation of this construction within string theory. As was already discussed in the introduction, open strings (with unspecified boundary conditions) lead to the differential operators as the crude approximation of the open string algebra — only end points of the open string are taken into account. The real picture is obviously more difficult and the first correction is that we should represent an open string state not as an interval but as a "slice of pizza" part of a closed string world-sheet [41, 42]. It is defined in the first approximation by *three points* — two of them are "ends" of the open string and the third one which is at the "apex" of the slice is responsible for the interaction with closed string vertexes. The description we encounter in the last section is very close to this qualitative picture. Curiously enough the very rough approximation of the open string algebra leads to a satisfactory description of the couplings of the open strings to RR gauge fields. Let us note at the end that the considerations of this paper, though inspired by string field theory, were rather formal. This allows us to present the logic behind the construction in the most explicit way. The detailed comparison/derivation of these results in terms of the first quantized string theory will be given elsewhere. It is also interesting to construct the natural "geometric" action for the algebra of differential operators in the lines of [43].

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